VTŠ: Osnovi računarske tehnike

Bulova algebra

mr. Veličković Zoran

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Bulova algebra

- Oko 1850, Britanski matematičar **George Boole** (1815–1864), je razvio novu formu matematike koja je poznata kao *Bulova algebra*.
- Bulova algebra je jedan od alata koji se koriste u dizajniranju elektronskih kola od kojih su sačinjeni računarski sitemi.
- Osnovna načela Bulove algebre su:
  - Logički iskaz može imati samo **tačnu** (TRUE) ili **netačnu** vrednost (FALSE).
  - Logički iskazi se mogu kombinovati na razne načine. Ako se iskazi kombinuju I (AND) operatorom nazivaju se **konjunkcije**.
  - Iskazi kombinovani ILI (OR) operatorom nazivaju se **disjunkcije**.
- Veza između logičkih iskaza može biti prikazana simboličkom logikom putem **jednačina** ili **tablicom istinitosti**.

Bulova algebra

Operatori u Bulovoj algebri

- Tek je Shannon, koncept Bulove agebre (TRUE-FALSE), primenio na binarne vrednosti **0** i **1**, koje se jednostavno mogu realizovati elektronskim kolima.
- Logičke funkcije se mogu predstaviti **grafičkim simbolima**, **jednačinama** ili **tablicom istinitosti**.
- Simboli **&**, **|**, **^** i **-** se koriste da predstave:
  - I (AND),
  - ILI (OR),
  - ISKLJUČIVO ILI (XOR)
  - NEGACIJU (NOT), (horizontalna linija).

Prikaz osnovnih logičkih funkcija

jednačina

y = a

y = a & b

y = a | b

y = a ^ b

Tablica istinitosti

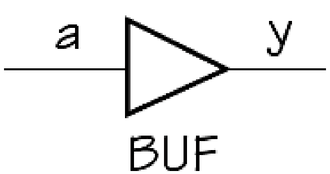
a	y
0	0
1	1

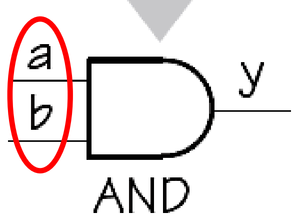
a	b	y
0	0	0
0	1	0
1	0	0
1	1	1

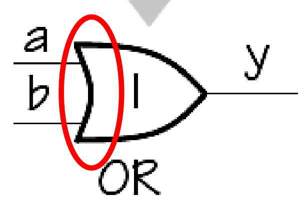
a	b	y
0	0	0
0	1	1
1	0	1
1	1	1

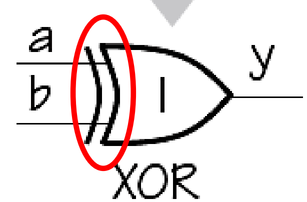
a	b	y
0	0	0
0	1	1
1	0	1
1	1	0

Grafički-Logičkim kolima









Alternativa

y=a

y=a·b

y=a+b

y=a⊕b

Prikaz osnovnih logičkih funkcija (2)

jednačina

$y = \bar{a}$ | $y = \overline{a \,\&\, b}$ | $y = \overline{a \mid b}$ | $y = \overline{a \wedge b}$

Tablica istinitosti

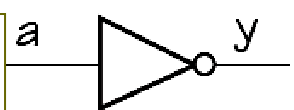
a	y
0	1
1	0

a	b	y
0	0	1
0	1	1
1	0	1
1	1	0

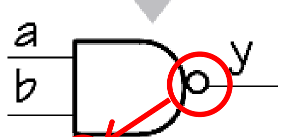
a	b	y
0	0	1
0	1	0
1	0	0
1	1	0

a	b	y
0	0	1
0	1	0
1	0	0
1	1	1

Logičkim kolima



NOT



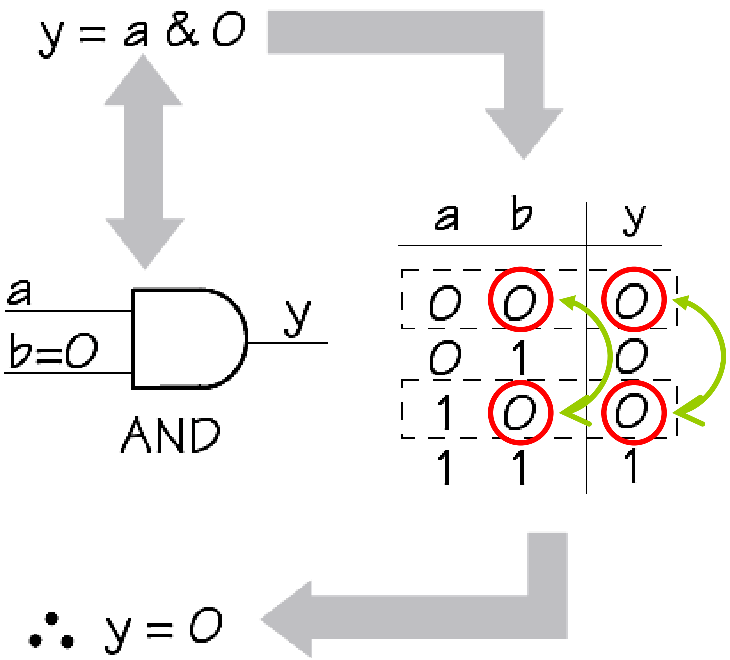
NAND

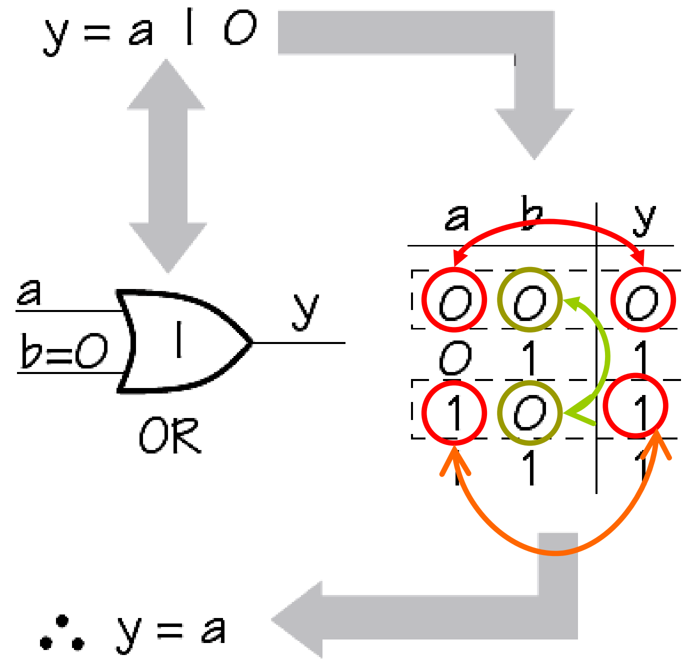
NOR

XNOR

Alternativa | **y=a′**

Specijalni slučajevi funkcija (1)





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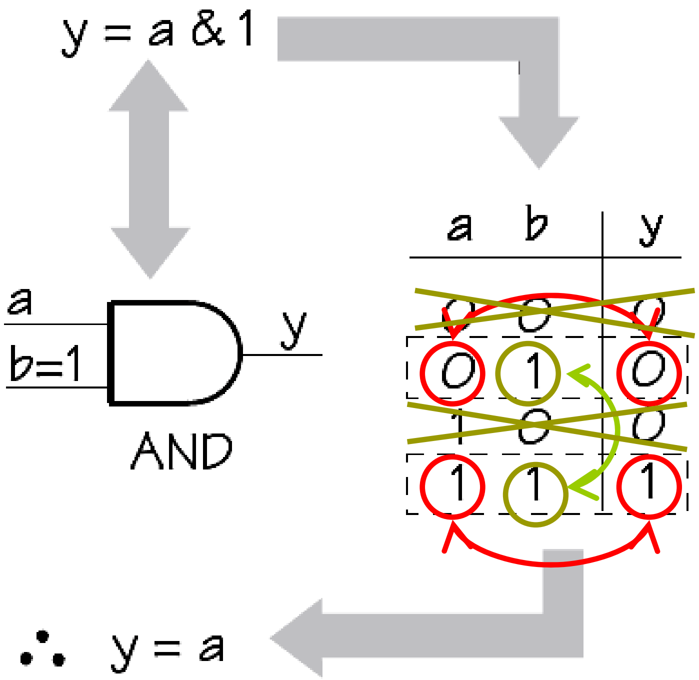
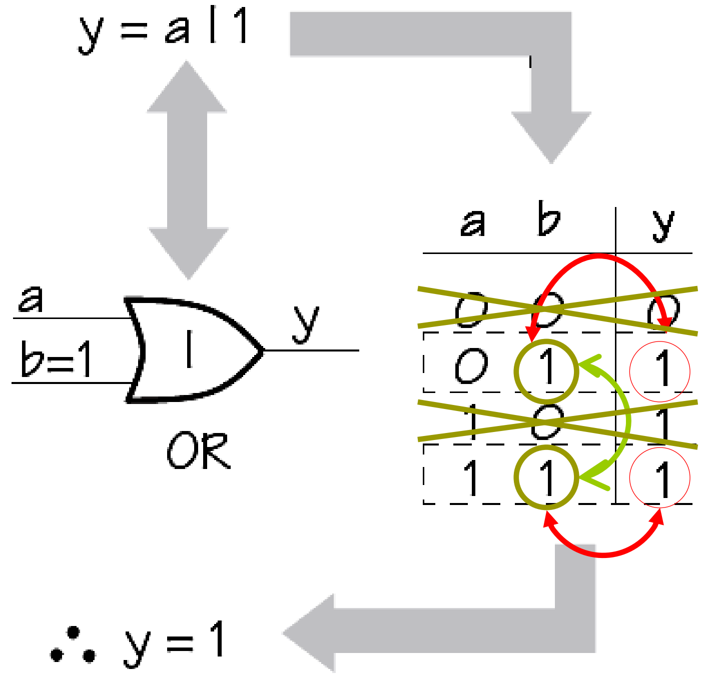
Specijalni slučajevi funkcija (2)





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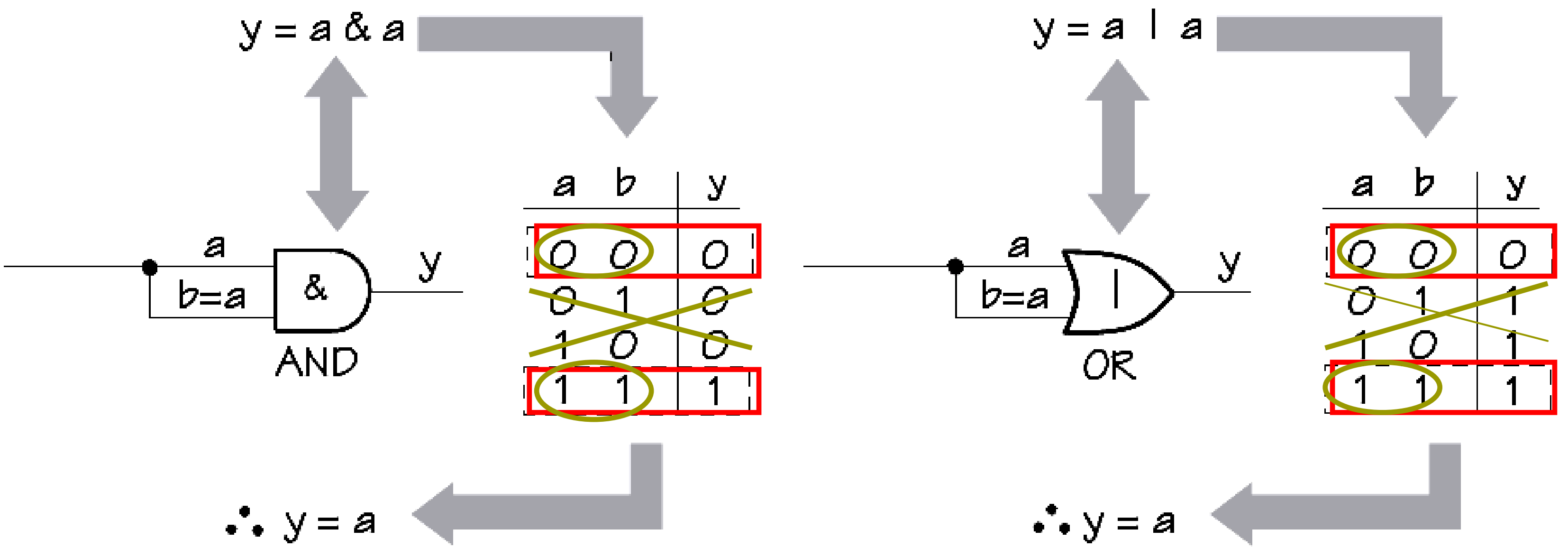
Specijalni slučajevi funkcija (3)



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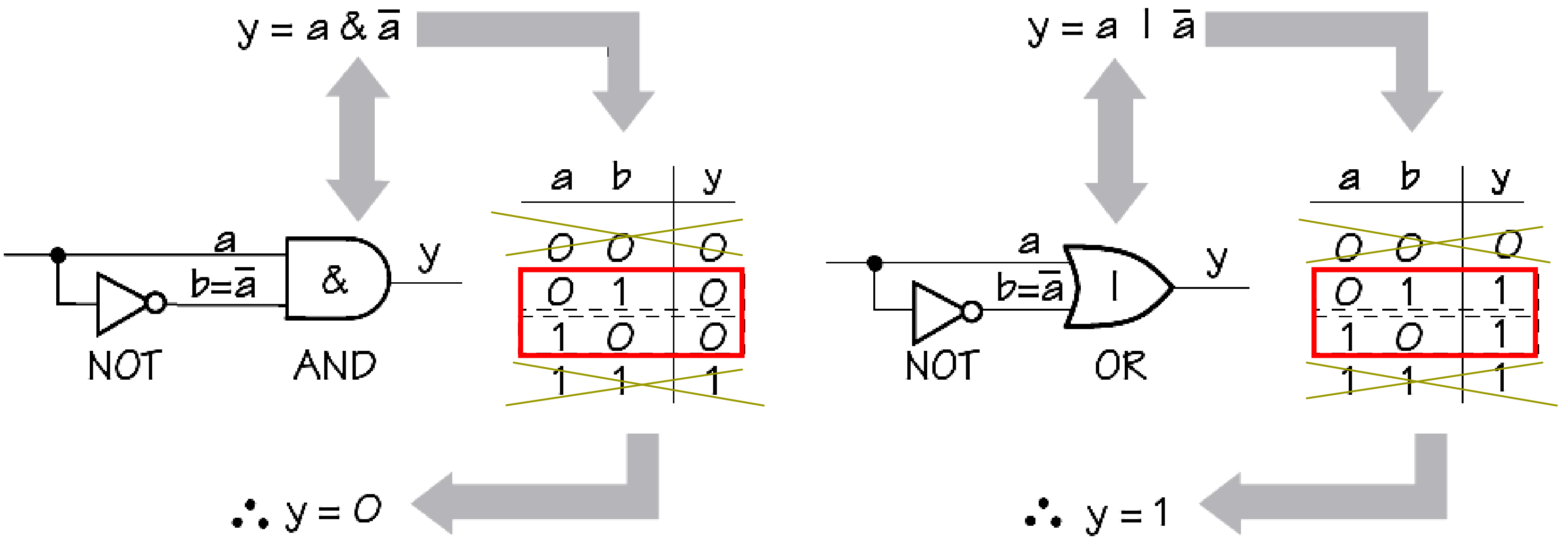
Komplementiranje



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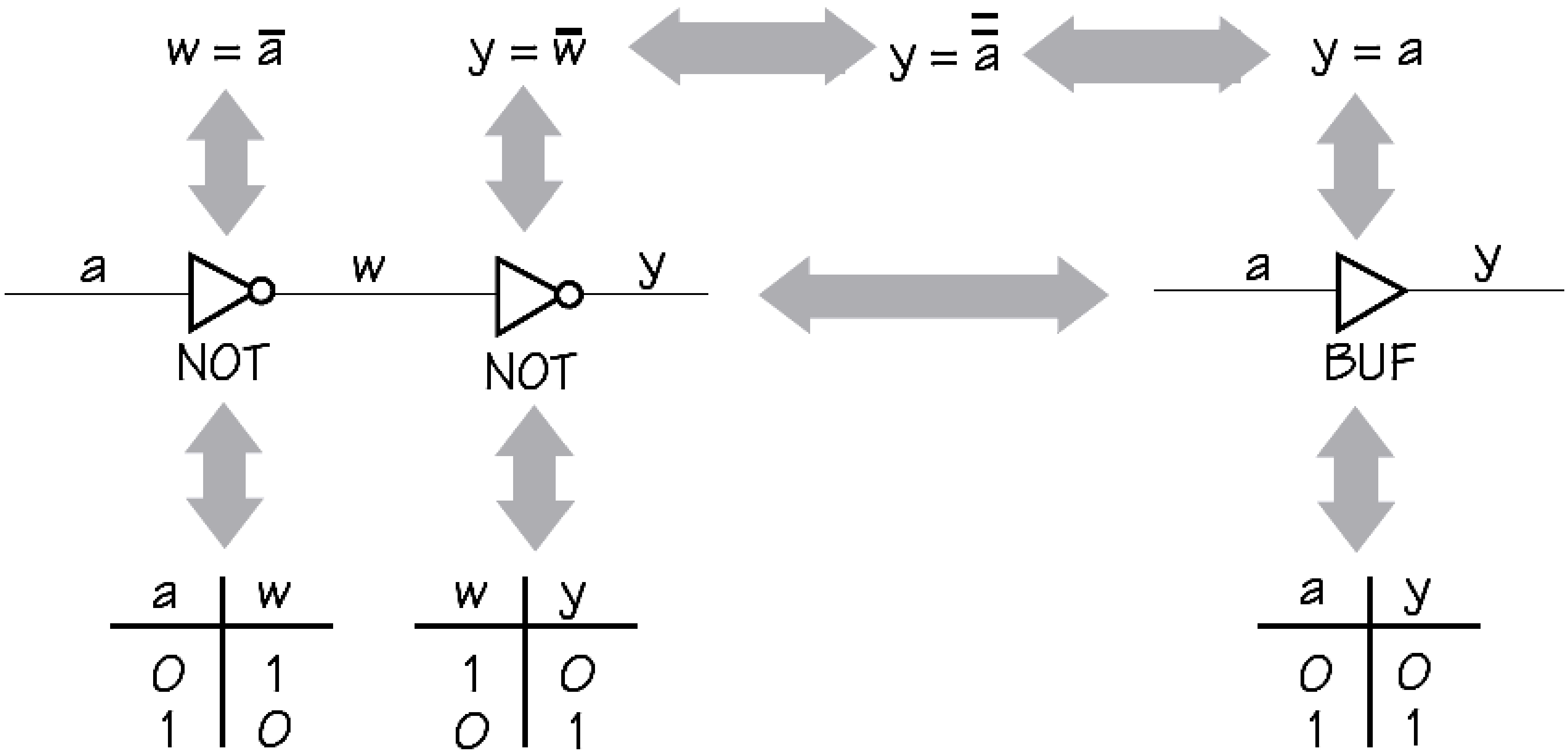
Negacija



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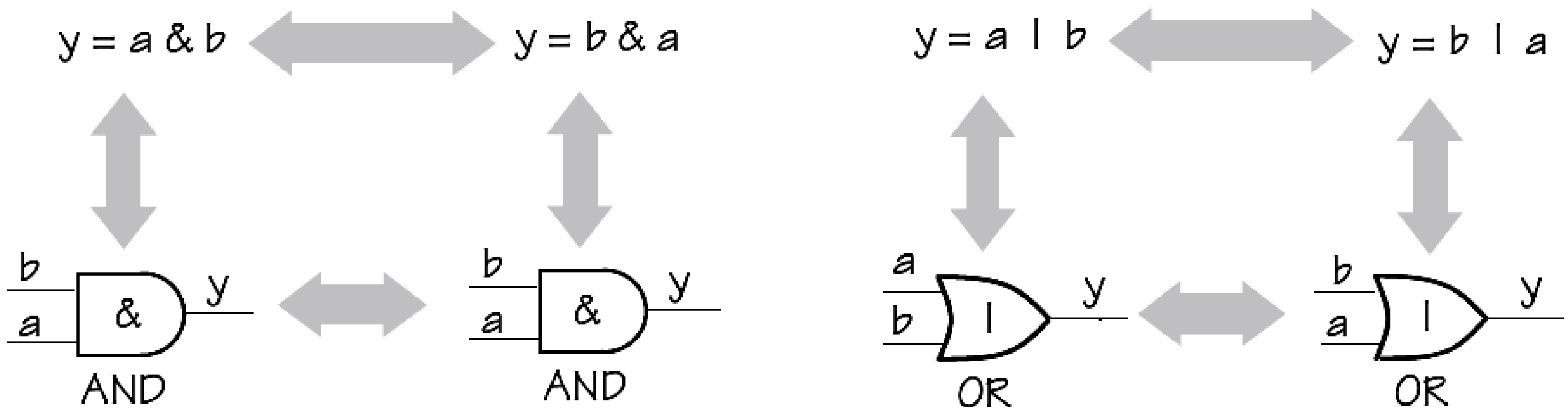
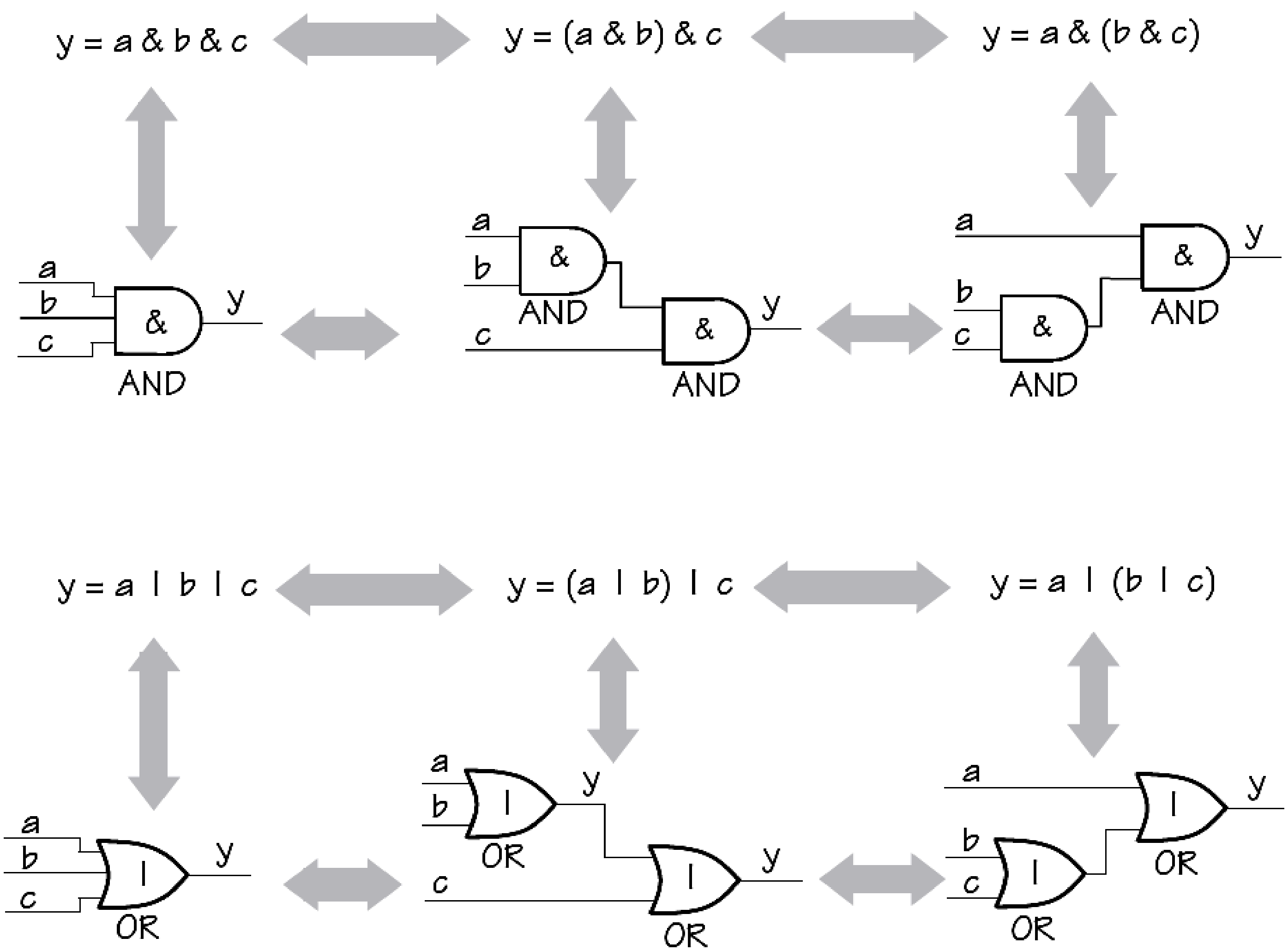
Osobina komutacije



Osobina asocijativnosti



Osnovne jedakosti u Bulovoj algebri

Table 13.11 Rules of Boolean algebra

1. $0 + X = X$	
2. $1 + X = 1$	
3. $X + X = X$	
4. $X + \overline{X} = 1$	
5. $0 \cdot X = 0$	
6. $1 \cdot X = X$	
7. $X \cdot X = X$	
8. $X \cdot \overline{X} = 0$	
9. $\overline{\overline{X}} = X$	
10. $X + Y = Y + X$	Commutative law
11. $X \cdot Y = Y \cdot X$	
12. $X + (Y + Z) = (X + Y) + Z$	Associative law
13. $X \cdot (Y \cdot Z) = (X \cdot Y) \cdot Z$	
14. $X \cdot (Y + Z) = X \cdot Y + X \cdot Z$	Distributive law
15. $X + X \cdot Z = X$	Absorption law
16. $X \cdot (X + Y) = X$	
17. $(X + Y) \cdot (X + Z) = X + Y \cdot Z$	
18. $X + \overline{X} \cdot Y = X + Y$	
19. $X \cdot Y + Y \cdot Z + \overline{X} \cdot Z = X \cdot Y + \overline{X} \cdot Z$	

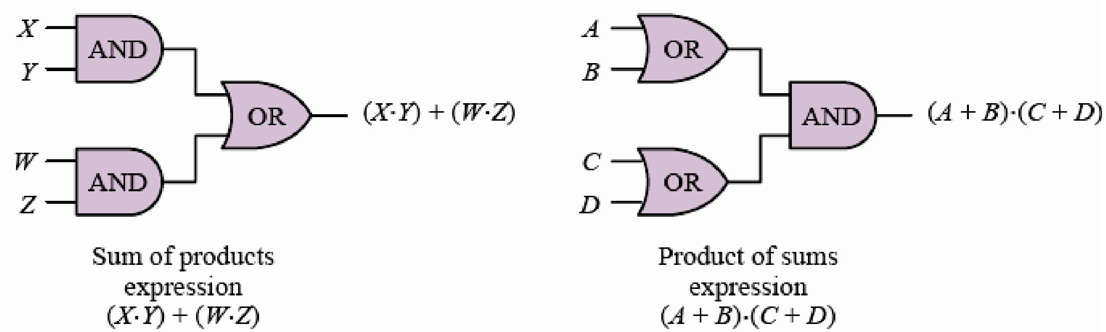


Figure 13.18 Sum-of-products and product-of-sums logic functions

Uprošćavanje log. iskaza

Problem

Using the rules of Table 13.11, simplify the following function using the rules of :
algebra.

$$f(A, B, C, D) = \overline{A} \cdot \overline{B} \cdot D + \overline{A} \cdot B \cdot D + B \cdot C \cdot D + A \cdot C \cdot D$$

Solution

Find: Simplified expression for logical function of four variables.

Analysis:

$$\begin{aligned} f &= \overline{A} \cdot \overline{B} \cdot D + \overline{A} \cdot B \cdot D + B \cdot C \cdot D + A \cdot C \cdot D & \\ &= \overline{A} \cdot D \cdot \left(\overline{B} + B\right) + B \cdot C \cdot D + A \cdot C \cdot D & \text{Rule 14} \\ &= \overline{A} \cdot D + B \cdot C \cdot D + A \cdot C \cdot D & \text{Rule 4} \\ &= \left(\overline{A} + A \cdot C\right) \cdot D + B \cdot C \cdot D & \text{Rule 14} \\ &= \left(\overline{A} + C\right) \cdot D + B \cdot C \cdot D & \text{Rule 18} \\ &= \overline{A} \cdot D + C \cdot D + B \cdot C \cdot D & \text{Rule 14} \\ &= \overline{A} \cdot D + C \cdot D \cdot (1 + B) & \text{Rule 14} \\ &= \overline{A} \cdot D + C \cdot D = \left(\overline{A} + C\right) \cdot D & \text{Rules 2 and 6} \end{aligned}$$

Prioritet Bulovih operatora

- Prioritet operatora u Bulovoj algebri je sličan kao kod standardne aritmetike:

$$6 + 2 \times 4 \equiv 6 + (2 \times 4)$$

$$a \mid b \,\&\, c \equiv a \mid (b \,\&\, c)$$

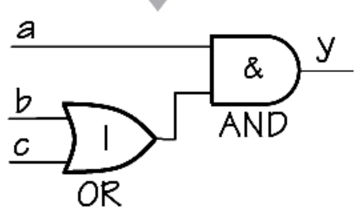
- Usled ove sličnosti & (AND) operator se naziva ***logičko množenje*** *ili proizvod, dok je* | (OR) operator poznat kao ***logičko sabiranje*** *ili suma*.

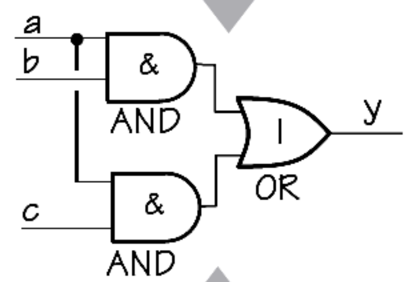
$$6 \times (5 + 2) \equiv (6 \times 5) + (6 \times 2)$$

Logičko (Bulovo) množenje

y = a & (b | c)

y = (a & b) | (a & c)





a	b	c	(b | c)	y
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

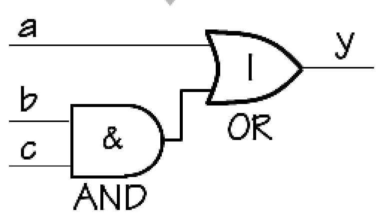
a	b	c	(a & b)	(a & c)	y
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	1	1
1	1	0	1	0	1
1	1	1	1	1	1

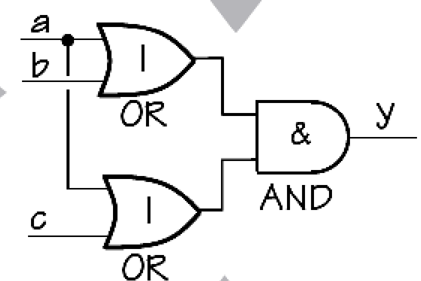
y kolone su identične

Prednosti

$y = a \mid (b \,\&\, c)$

$y = (a \mid b) \,\&\, (a \mid c)$



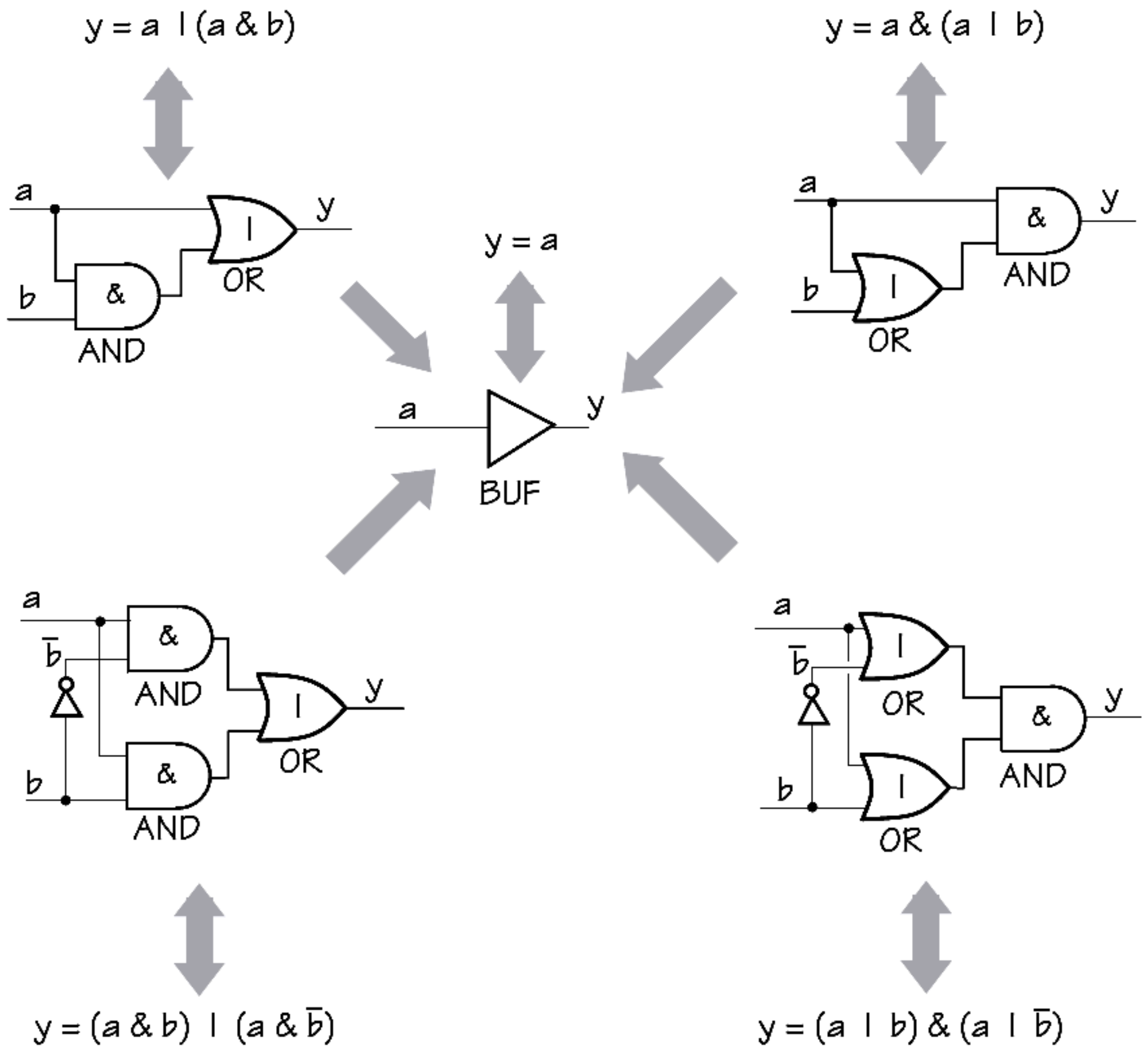


a	b	c	(b & c)	y
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

a	b	c	(a | b)	(a | c)	y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	1	1	1
1	1	1	1	1	1

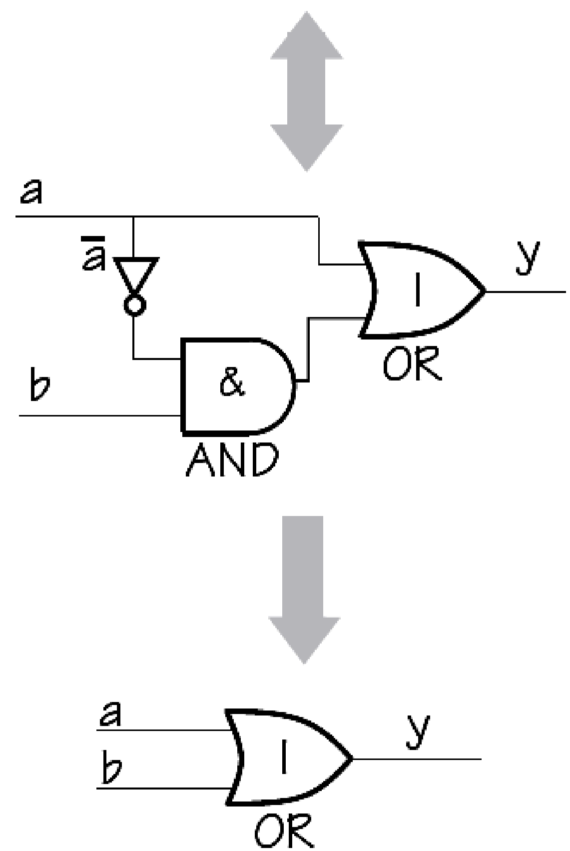
Y kolone su identične

Spec. slučajevi logičkog sabiranja



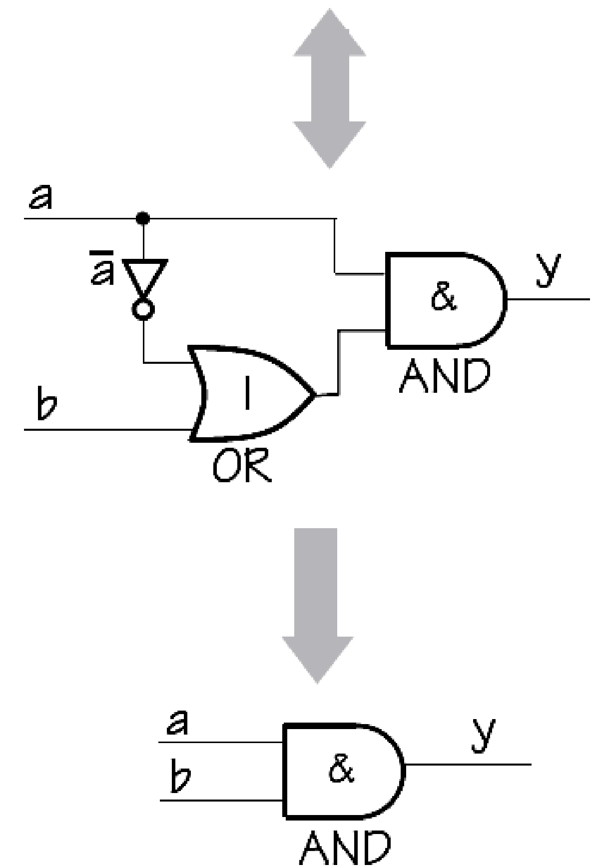
Uprošćavanje log. izraza

$y = a \mid (\bar{a} \,\&\, b)$



y=a | b

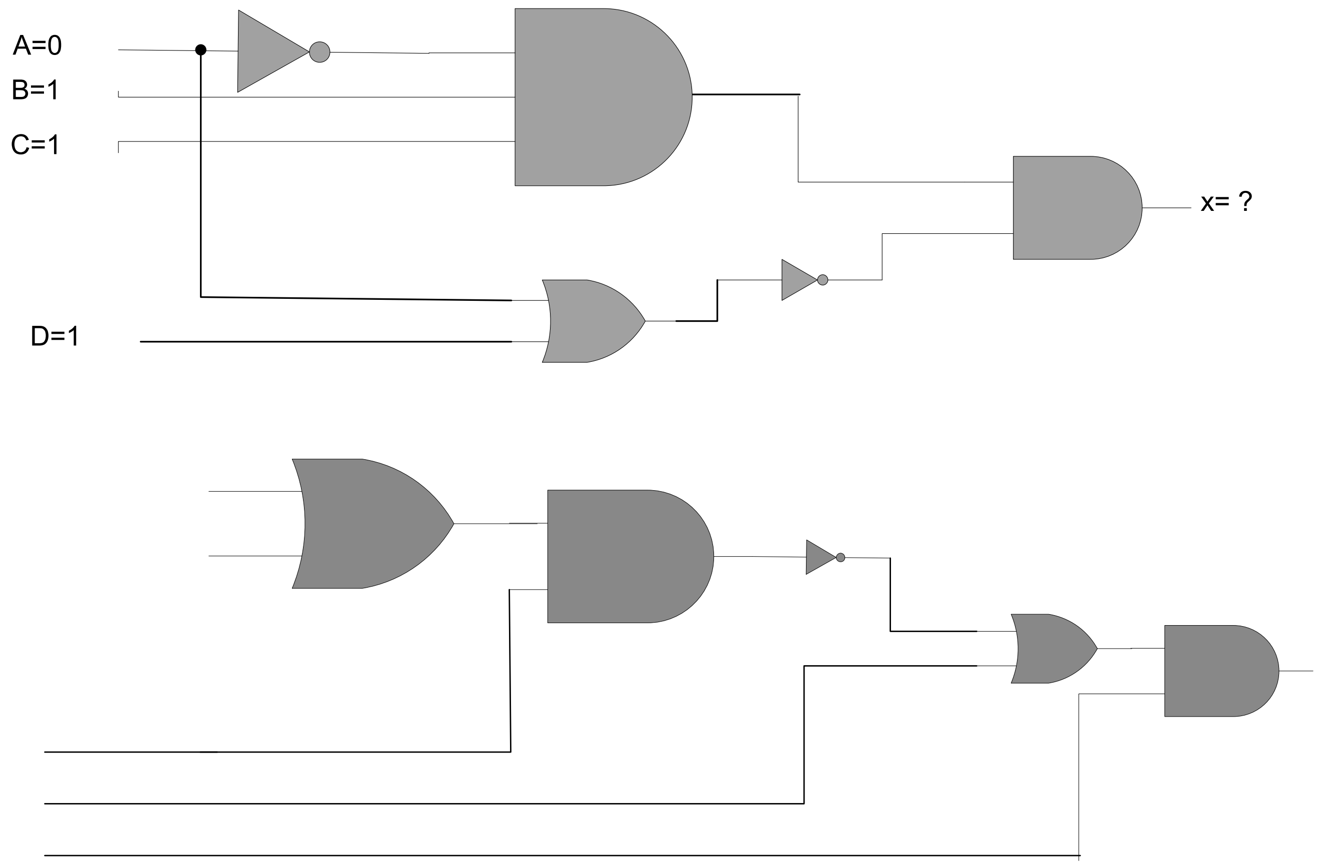
$y = a \,\&\, (\bar{a} \mid b)$



y=a + b

Domaći zadatak

- Odrediti logičko stanje x na izlazu.
- Napisati logičke funkcije datih kola.

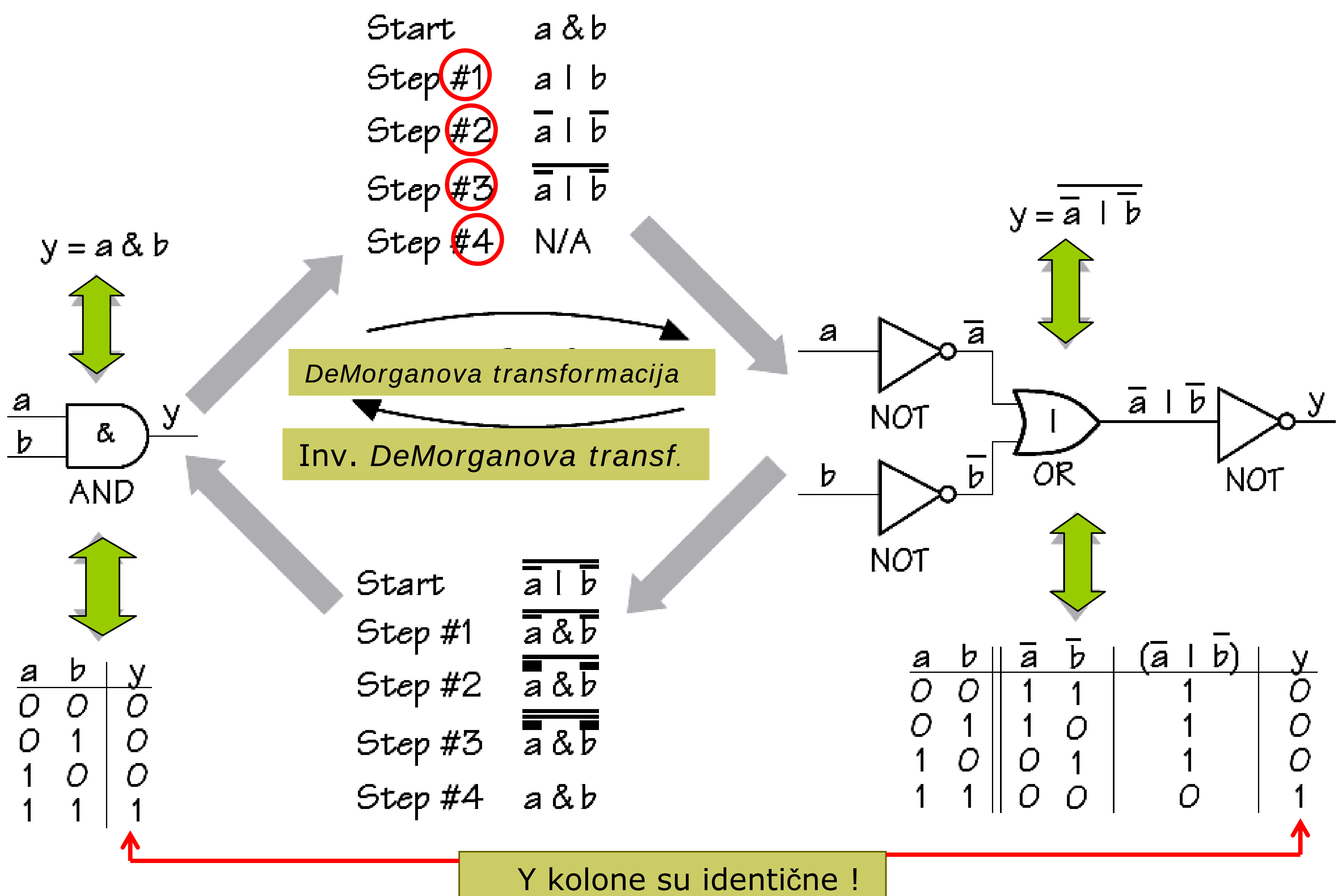


DeMorganovi obrasci

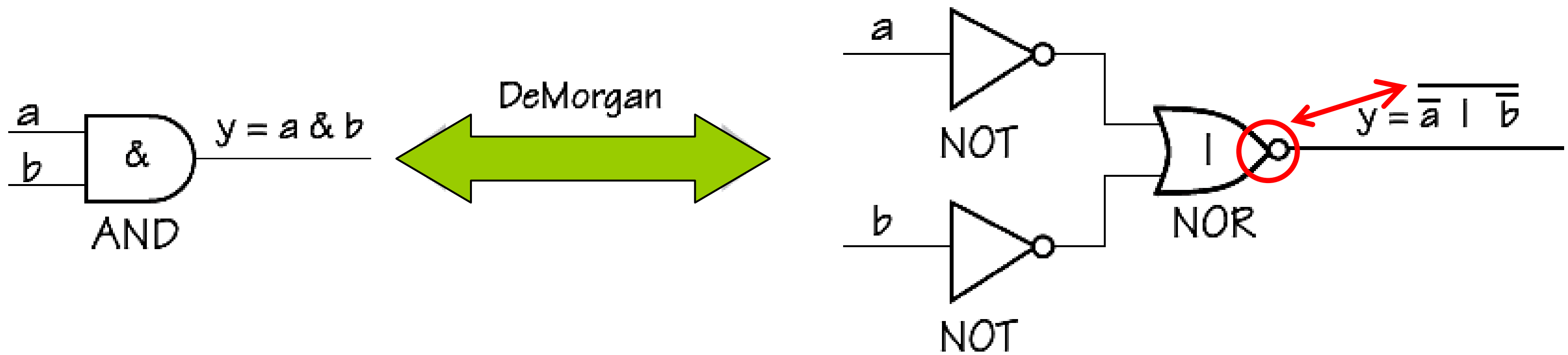
- *DeMorgano-vi* obrasci **transformišu logičke funkcije**.
- *DeMorgan-ove transformacije se satoje od* ***4 koraka*** (stepa):
  1. Izmenite sve **&** operatore sa **|** operatorima i obratno.
  2. Invertujte sve promenljive, takođe izmenite **0** u **1** i obratno.
  3. Invertujte **kompletnu** funkciju.
  4. Redukujte niz **više invertora** (ako postoji).
- Pogledamo primer prmene *DeMorganovih obrazaca* na sledećem slajdu:

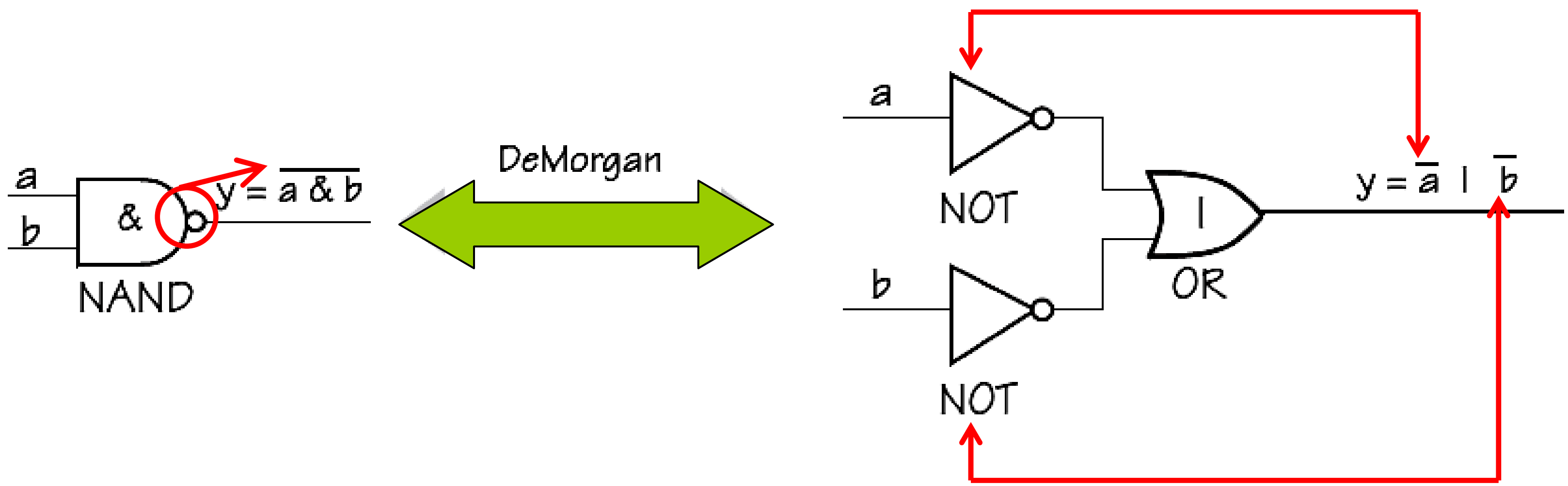
y=a & b

2-ulazna I funkcija (1)

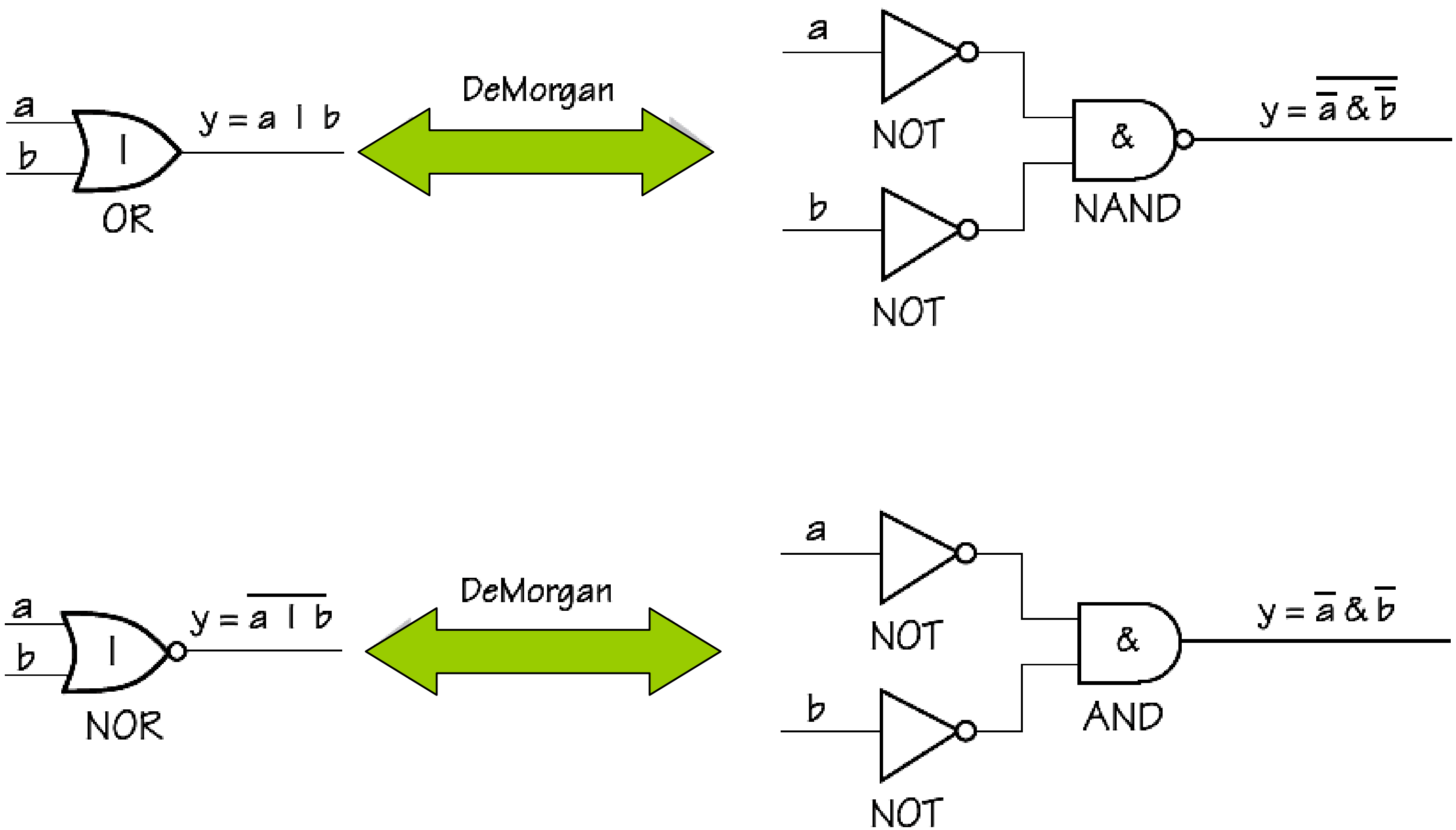


2-ulazna I/NILI funkcija (2)





2-ulazna ILI/NILI funkcija



Realizacija Bulovih funkcija

- U osnovi postoje dve tehnike za realizaciju Bulovih jednačina direktno iz tabele istinitosti.
- Kod prve tehnike formiraju se **mintermi** za svaku liniji iz tabeli istinitosti čiji je izlaz jednak **1** a zatim se povezuju **ILI** operaterom.
- Ova realizacija se naziva ***suma-proizvoda***.
- Kod druge tehnike formiraju se **maxtermi** za svaku liniji iz tabeli istinitosti čiji je izlaz jednak **0** a zatim se povezuju **I** operaterom.
- Ova realizacija se naziva ***proizvod-suma***.

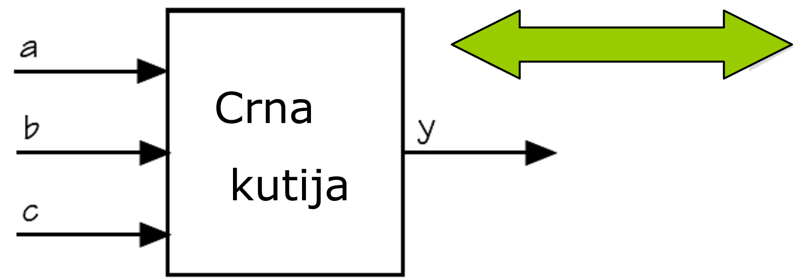
Kanonične forme: minterm, maxterm

a	b	c	mintermi	maxtermi
0	0	0	$(\bar{a} \& \bar{b} \& \bar{c})$	$(a \mid b \mid c)$
0	0	1	$(\bar{a} \& \bar{b} \& c)$	$(a \mid b \mid \bar{c})$
0	1	0	$(\bar{a} \& b \& \bar{c})$	$(a \mid \bar{b} \mid c)$
0	1	1	$(\bar{a} \& b \& c)$	$(a \mid \bar{b} \mid \bar{c})$
1	0	0	$(a \& \bar{b} \& \bar{c})$	$(\bar{a} \mid b \mid c)$
1	0	1	$(a \& \bar{b} \& c)$	$(\bar{a} \mid b \mid \bar{c})$
1	1	0	$(a \& b \& \bar{c})$	$(\bar{a} \mid \bar{b} \mid c)$
1	1	1	$(a \& b \& c)$	$(\bar{a} \mid \bar{b} \mid \bar{c})$

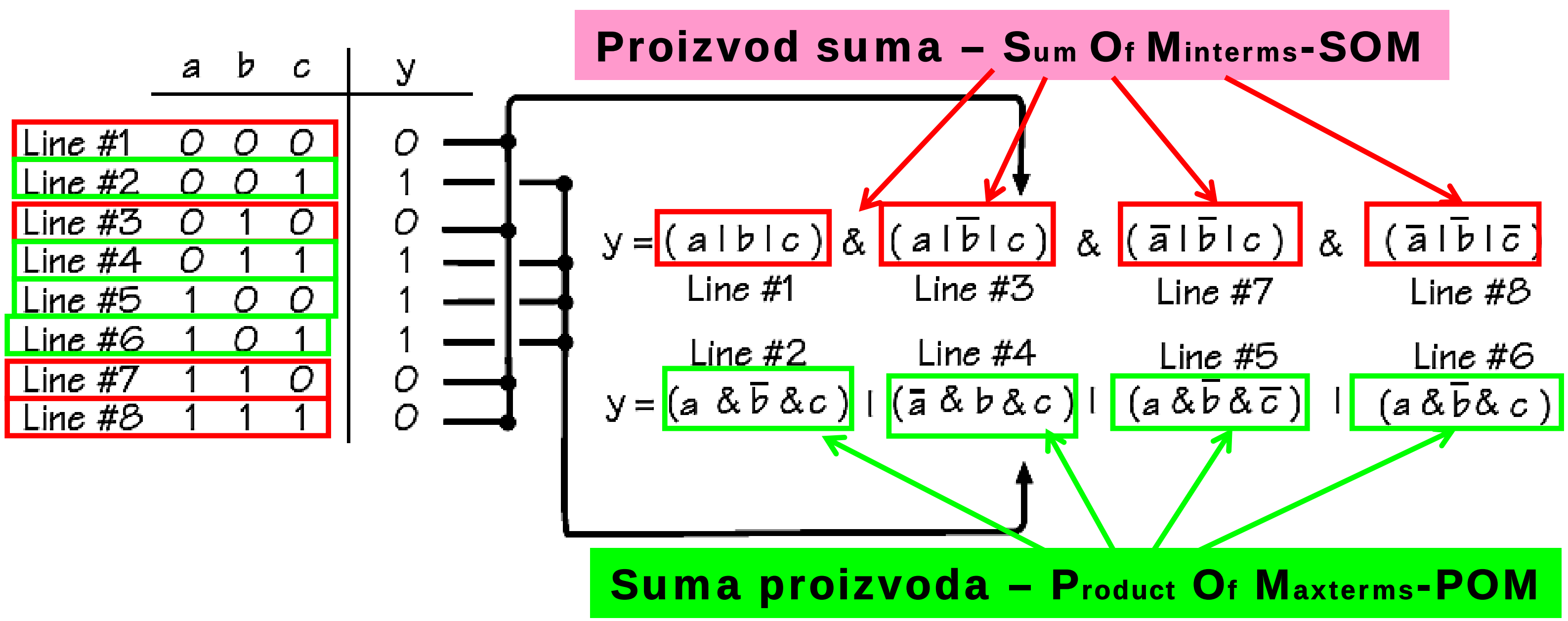
"I" funkcije

"ILI" funkcije

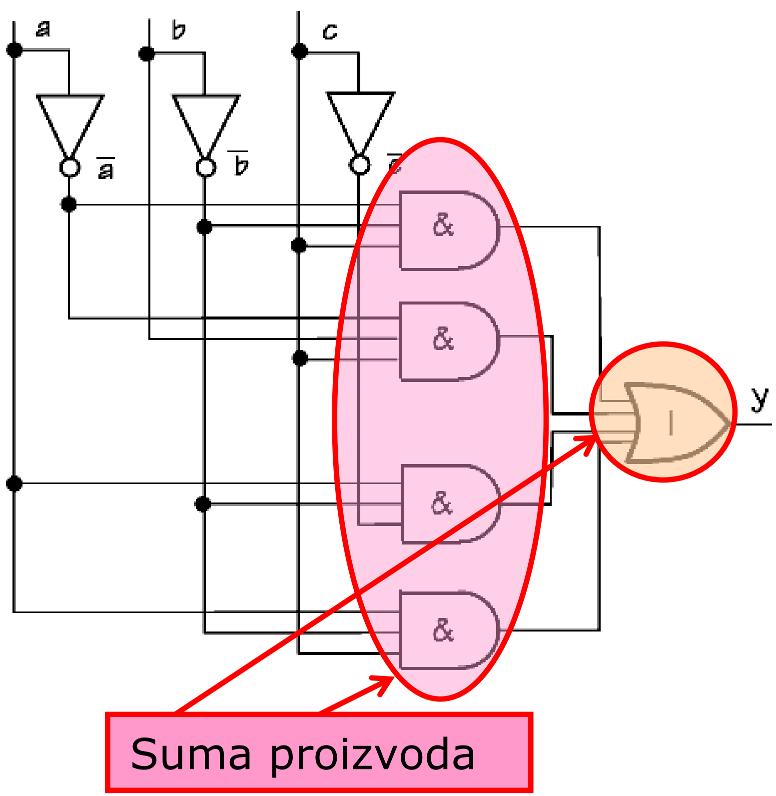
Dizajn prekidačkih funkcija

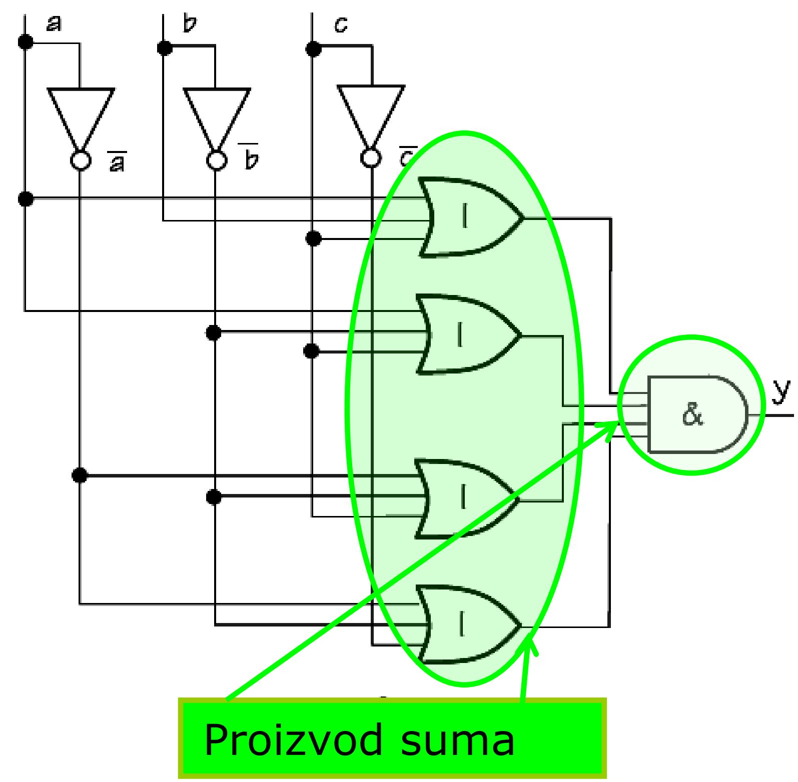


a	b	c	y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0



Realizacija prekidačkih funkcija





Domaći zadatak

- Realizujte funkciju y predstavljenu tablicom istinitosti:

A	*B*	*C*	*D*	*y*
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1